BIBLIOGRAPHY

ACOSTA, RODLYNN MAY M. AND NORMAN, CLEVENSON C. APRIL 2008. Analysis on the Number of Students Serviced at the Cashier's Counter. Benguet State University, La Trinidad, Benguet.

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ABSTRACT

This study was conducted to determine the arrival distribution of students and service time rendered by the cashier at the counter of Benguet State University Administration and the queue discipline of the students waiting at the cashier counter.

Poisson and exponential distribution were employed in this study. It has been found that the average number arrival at the cashier counter is 76 students per hour and the average number of students that the cashier counter can serviced per hour is 26. This discrepancy of student's arrival and number of students serviced at the cashier's counter resulted to long queues.

The model was identified to fit the enrollment operation with three cashier counters in the administration. The measuring capacity utilization (ρ) shows that 97.44% of time the counters are busy. The average time a student spent at the system is 0.5142 hours, which means that a customer will receive complete service for approximately 30.85 minutes including the time the students waiting in the line. The average time in the queue is 0.475738 hours which is approximately 28.54 minutes, which is the time spent

waiting in the line. The remaining time, which is W-Wq, is the time spends in the counter, 0.038462 hours or 2.30772 minutes.

To improve the operating characteristics of the queueing of students serviced at the cashier’s counter; some suggestions may be offered to the management to change the arrival rates in a number of ways, such as providing discounts or other incentives for the early enrollees. Adding one to the existing number of cashier counter will help reduce number of students waiting in long queues. Results shows that if we add another counter, the average number of students in the system was reduced from 39 to 14 students per hour.; the average time students spends in the system is reduced from 30.8 minutes to 10.75 minutes and the probability that an arriving student has to wait for service was reduced from 95% to 39%. More counter, the better the service.

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INTRODUCTION

Background of the Study

Waiting in lines is part of everyday life. Whether it is waiting in line at a grocery store to buy items or checking out at the cash registers, or waiting at an amusement park to go on the newest ride, a lot of time waiting is spent. People, wait in lines at the movies, campus dining rooms, the Registrar's Office for class registration, and many more.

Most people hate waiting. But reduction of the waiting time usually requires extra investments. To decide whether or not to invest, it is important to know the effect of the investment on the waiting time. So models and techniques are needed to analyze such situations according to Mohr (1983).

Medhi (1991) in his Stochastic Models in Queueing System, states that attention is paid to methods for the analysis and applications of queueing models. It is particularly useful for the design of these systems in terms of layout, capacities and control.

Cox and Smith (1973) on "Queues" write that queueing model is used to approximate a real queueing situation or system, It is characterized by arrival process of customers, behavior of customers,service times and discipline. Arrival process of customers usually assumes that the interarrival times are independent and have a common distribution. In many practical situations customers arrive according to a Poisson stream (i.e., exponential interarrival times). Customers may arrive one by one, or in batches. Second is the behavior of customers. They may be patient and willing to wait for a long time or customers maybe impatient and leave after a while. Third is the service time which usually assumes that the service times are independent and identically distributed, and that they are independent of the interarrival times. It can also occur that service times are dependent of the queue length. For example, the processing rates of the machines in a production system can be increased once the number of jobs waiting to be processed becomes too large. Fourth is the service discipline. Customers can be served one by one or in batches. There are many possibilities for the order in which they enter service." First come, first served", in the order of arrival wherein the first one who arrived is the one who will be entertained first like in the paying of fees especially in the enrollment of the students.

Upon entering school, students must first enroll. Considering the enrollment procedure of the College of Arts and Sciences students, students must present their class card to the teacher who is assigned to assess the students to be recorded at the checklist. After presenting the grades, the students will be given an enrollment form to be filled up before assessment. The students will then go to the Dean's office to be assessed for their fees and then proceed to the Administration building for paying. There, students wait for their turn to be entertained. However, there are only three (3) counters who will render the sevice in the Administration. It is expected that a line will be formed. This is a system where students arrive, join a waiting line, wait their turn, serve by a multiple server and depart. It is described as the queueing system.

Objectives of the Study

This study aimed the following:

1. To determine the arrival distribution of students at the cashiers counter of BSU Administration;

2. To determine the service time rendered by the cashier to the students at the counter;
3. To determine the queueing model and operating characteristics of the student enrollees at the cashier's counter; and
4. To determine the economic analysis of the operating characteristics of queueing of students enrollees at BSU Admistration.

Importance of the Study

The result of the study provides information to the management on the proper management of queue and on what level of services will they offer. The arrival rate specifies the average number of students arriving at the cashier's counter per hour which could help the management decide whether or not to make changes or choose alternatives to improve the situation that involves waiting lines.

The study also reminds students, to practice and develop the value of patience and willingness to wait for their turn.

The result of the study serves as a basis for the other researchers who will be interested to conduct queueing analysis.

Scope and Delimitation

The study was conducted at Benguet State University Administration Building, La Trinidad, Benguet, during enrollment period for the Second semester school year, 2007-2008. The data gathering was executed on October 22-26, 2008. The main focus of the study is the application of queueing analysis of the students serviced at the cashiers counter regarding paying their enrolment fees.

REVIEW OF RELATED LITERATURE

Queue is everywhere. It is seen everywhere, paying for bills, line at a supermarket counter, line at the banks machine, line of cars at a traffic light show. Waiting line is undesirable for all parties times, considering the following studies concerning queues.

<u>Studies on Queueing</u>

Mohr (1983) cited that Arrival Distribution for a waiting line involved determining how many costumers arrive in a period of time. He determined the number of customers in a 1-hour period, since the number of customer each hour is not necessarily constant.

Queueing analysis is applied also on the parallell simulation of queueing networks as studied by Sun, Z. et. al. (1991). Its limitations and potentials in performace evaluation review on the queueing delay and cell loss for combined traffic sources in ATM networks in United Kingdom as the main purpose. The development of broadband ISDN based on the ATM introduces new network characteristics different to those traditional channel based networks, typically queueing delay and cell loss when combined traffic sources share the same bandwidth capacity. The paper reports the work on studying the new network characteristics so that the bandwidth resource can be utilized efficiently with restricted queueing delay and cell loss.

Another application of queueing was undertaken by Denisov and Sapozhnikov (2006) of the Department of Mathematics/Boole Centre for Research in Informatics, University College Cork, Cork, Ireland on the distribution of the number of customers in the symmetric M/G/1 queue wherein they consider an M/G/1 queue with symmetric service discipline. In this research they show that this conjecture is true if service requirements have an Erlang distribution. They also show by a counter example, involving the hyper exponential distribution, that the conjecture is generally not true.

In Madrid, Spain, Artalejo, Economou and Lopez-Herrero (1985) of the Department of Statistics used queueing model on their research entitled “Analysis of a Multiserver Queue with Setup Times” that deals with the analysis of an M/M/c queueing system with setup times. This queueing model captures the major characteristics of phenomena occurring in production when the system consists in a set of machines monitored by a single operator, which carry out an extensive analysis of the system including limiting distribution of the system state, waiting time analysis, busy period and maximum queue length.

Beaubrun (2007) analyzes the traffic distribution and blocking probability in future wireless networks through Poisson and Exponential distribution. Traditional analysis of teletraffic in such networks assumes that call arrivals follow a Poisson process, as each cell is being modeled as an M/G/c/c queueing system. Such a model has not been explicitly addressed in the literature, the main contribution is to propose a solution which enables to evaluate both traffic distribution and blocking probability within each cell of the service area. Result analysis reveals that coefficient of variation of call arrivals has more impact on the network performance than coefficient of variation of channel holding time.

Wei (2003) conducted a study on grids. Grid computing has emerged as an important new field, distinguished from conventional distributed computing by its focus on large-scale resource sharing, innovative applications, and high-performance orientation. A Grid integrates and coordinates resources and users that live within different control domains with the goal of delivering non-trivial quality of service (QoS). Consequently, task management and scheduling is of central importance for the Grid-based systems. In this paper, the task dispatching and selecting on the distributed Grid computing system by a multiserver multiqueue (MSMQ) model, and propose a modeling and analysis technique based on Stochastic Petri Net (SPN) methods are used. An approximate analysis technique is also proposed to reduce the complexity of the model.

Queues in the Philippines are common scenery and queueing discipline is not that practiced. Thus, this study is conducted regarding long lines. However, the application of queueing analysis here in the Philippines is not limited and the utilization is not yet fully explored. It is due to lack of device or strategy in recording the data necessary for the analysis and even the availability in some packages, is limited.

THEORETICAL FRAMEWORK

Cox and Smith (1993) state that queueing theory deals with mathematical models of various kinds of real queues, example is the situations where congestion occurs due to randomness in arrival and service times and customers have to wait for service. Queues occur when the current demand for service exceeds the capacity of service facilities, and the main purposes of the theory is to provide means for designing/modifying/optimizing service systems in such a way as to reduce the likelihood of queues, the customer's waiting times and so on. Analysis of queueing models may include determining the distributions of the queue length, waiting times and the duration of the busy/idle periods for servers.

Trivedi (1982) states that a queue or queueing system is a system which includes a random input stream of requests which needs service and a mechanism which provides that service. Typical examples of queues are telephones exchanges, customer's queues at checkout counters on business institutions in an airport, and a network of time sharing computers.

Since cashiers counter of Benguet State University Administration Building consist of a three queue and three service facility and assumes that arrivals are unlimited and can be formally treated in a Poisson distribution. Service times are assumed to take on the form of an Exponential distribution.

Arrival Distribution

For many waiting lines, the arrivals occurring in a given period of time appear to have a random pattern-that is, while there may be a good estimate of the total number of arrivals expected, each arrival is independent of other arrivals and cannot predict when the arrival will occur. In this case, Poisson distribution will be used because it provides a good description of the arrival pattern.

The Poisson probability distribution is defined as:

$$P(x) = \frac{\lambda^{x} e^{-\lambda}}{x!} \qquad \text{for } x = 0, 1, 2, \ldots$$

Where, in waiting line applications,

x = number of arrivals in a specified period of time

λ= average of expected number of arrivals for the specified period of time

e = 2.71828

Service Time Distribution

A service fine probability distribution is needed to describe how long it takes a student to wait for his turn and to pay. Since students have different amount of fees, cashier's service times vary. Thus, exponential probability distribution will be used.

The exponential probability distribution is defined as:

$$f(x) = \mu e^{-\mu x} \qquad \text{for } x \geq 0$$

where:

x = service time

μ = average or ecpected number of units that the service facility can handle in a specific period of time

e = 2.71828

Multiserver-Multiqueue Model

A logical extension of the single-channel waiting line is the multiserver multiqueue single channel waiting line. By multiple-channel waiting lines we mean that two or more counter or service locations are present. It allows us to examine the situation where the number of counters in service facility may assume any finite value

To do this, the following short-hand notation to solve the formula given is used.

Where:

λ = average arrival rate of the students

μ = average service rate per counter

s = number of available counter

1. the percentage of time the counter are busy or the probability that the counters are busy

$$\rho = \frac{\lambda}{s\mu} \qquad \text{for } s\mu > \lambda$$

2. the probability that no students in the ounter

$$Po = \frac{1}{\frac{\sum \frac{\lambda^n}{\mu}}{n!} + \frac{\frac{\lambda^s}{\mu}}{s!(1-(\rho/s))}}$$

3. the average time a student spent in the queue waiting for the service

$$Wq = \frac{\dfrac{\rho^S Po}{s!(1-(\rho/s))}}{s\mu(1-(\rho/s))}$$

4. the average time a student spends in the system (waiting time + service time)

$$W = Wq + \frac{1}{\mu}$$

5. the average number of students in the queue waiting for service

$$Lq = \lambda Wq$$

6. the average number of students in the service system

$$L = \lambda W$$

Definition of Terms

The following are defined for the purpose of the study:

Arrival defines the way customers enter the system. Mostly the arrivals are random with random intervals between two adjacent arrivals. Typically the arrival is described by a random distribution of intervals also called Arrival Pattern.

Arrival Rate is the average number of students arriving per time period denoted by lambda.

Cashiers counter is a window where cashier's stay and does its work.

Exponential distributions are a class of continuous probability distribution. They are often used to model the time between independent events that happen at a constant average rate.

Poisson distribution is a discrete probability distribution that expresses the probability of a number of events occurring in a fixed period of time if these events occur with a known average rate, and are independent of the time since the last event.

Queue is also known as the waiting line, that is, the place where a certain number of customers spend most of their time waiting for service in the queueing system.

Queueing discipline represents the way the queue is organized.

Queueing system considered to be the waiting lines or the queues as well as the number of available service facility which contains one or more server.

Service Time, as applied to queuing systems, is a period of time describing how long it takes a student to be serviced.

METHODOLOGY

Locale and Time of the Study

The study was conducted at Benguet State University, La Trinidad, Benguet during the second semester, school year 2007-2008.

Data Gathering Tool

Data were gathered through individual observations among the Benguet State University students who paid their fees during enrollment period at the cashiers counter (Administration Building), on the 22nd until 26^{th} day of October 2007. The data gathered were the arrival time, waiting time and service time.

Data Analysis

The summarized data were utilized in the application of Poisson probability distribution to explain the arrival pattern of the Benguet State University students serviced at the Cashier's counter. Exponential probability distribution was also used to provide a good description of the service time distribution. The Multiserver-Multiqueue-Single-Channel Waiting line Model was utilized to determine the queue data characteristics.

The data gathered were encoded,summarized and were analyzed using Microsoft Excel. Average number of students arriving per hour (λ) and the average number of students that can be served per hour (μ) were computed using the Microsoft Excel.

RESULTS AND DISCUSSION

Arrival Distribution

Table 1.1 presents the summary for the arrivals of students per hour interval, the mean of the arrivals, the total number of arrivals and the overall mean.

Time interval 1:00-2:00 had the highest students arrival of 869 with the highest mean of 174 students arrivals per hour. This time interval had the highest arrival because students were done processing their enrollment and now ready to pay, where, time interval 4:01-5:00 had the lowest number of arrivals of 62 and lowest mean of 12. This is because the time interval that was considered was 30 minutes wherein cashiers are leaving the counters and the closing time for the administration.

The students arrivals were not scheduled in paying and occurred in an unpredictable manner, a random pattern appears to exist. Mohr (1983) determined that indeed, arrival is not constant in a period of time. Thus, to determine the student arrivals, the Poisson distribution provides a good description of the arrival pattern which is defined by as $P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$ for x=40, 42, 44,…,60

where λ= 76 and e=2.71828.

Table 1.1 Summary of arrivals and arrival means

HOUR INTERVAL	# OF STUDENTS	MEAN ARRIVAL/HOUR
8;00-9:00	97	19
9:01-10:00	440	88
10:01-11:00	467	93
11:01-12:00	232	46
1;00-2;00	869	174
2;01-3:00	558	112
3:01-4:00	310	62
4:01-5:00	62	12
overall mean		76

Table 1.2 presents the Poisson students arrival distribution of Administration cashier counter ,where we would expect 40 students arriving in an hour 1.75% of the time, exactly 45 arrival in an hour 4.11 % of the time, we would expect 50 students arriving in an hour 5.58% of the time, and so on.

The study by Beaubrun (2007) also uses the same distribtution we used in this study where in, he uses Poisson in assuming the call arrivals which its main contribution is to propose a solution which enables to evaluate both traffic distribution and blocking probability within each cell of the service area.

Table 1.2 Arrival distribution

X=NUMBER OF ARRIVALS	P(X≤X)
40	0.017464586
42	0.026379436
43	0.036264753
44	0.04556745
45	0.052535877
46	0.055773802
47	0.054701229
48	0.049712752
49	0.041981451
50	0.033028964
51	0.024267891
52	0.054701229
53	0.052637032
54	0.049712752
55	0.04609728
56	0.041981451
57	0.037562351
58	0.033028964
59	0.02855046
60	0.024267891

Service Time Distribution

Table 2.1 presents the summary for the number of students serviced by the cashier per one hour interval, the mean of the students serviced by the cashier, the total number of students who were serviced and the overall mean of the students being served by the cashier counter per day.

Time interval 1:00-2:00 had the highest mean students that were served by the cashier counter of 40 which was equal to the highest students arrival 869. This implies that the numbers of arrivals per hour was 76 and the number of students that was being served by the cashier counter per hour was 26. Thus there is the discrepancy between the arrivals of students per hour and the students serviced by the cashier counter per hour is the reason for queueing. Time interval 8:00-9:00 had the lowest number of students who were served for a reason that students are still processing their enrollment procedures.

Table 2.1 Summary of average students serviced per hour

HOUR INTERVAL	COUNTER I	COUNTER II	COUNTER III	MEAN SERVICED/HOUR
8;00-9:00	6	5	18	2
9:01-10:00	45	150	101	20
10:01-11:00	187	146	168	33
11:01-12:00	151	184	112	30
1;00-2;00	209	164	202	40
2;01-3:00	221	154	186	37
3:01-4:00	155	152	162	31
4:01-5:00	3	70	85	11
overall mean				26

The exponential probability distribution was used to describe the service time, since students demanded a different service that depends on what amount they are supposed to pay, thus the Cashiers counter service times varied. The exponential distribution is defined by $f(x) = \mu e^{-\mu t}$ for $x \geq 0$ where μ=26 and e=2.71828. Table 2.2 presents the exponential distribution for the service time thus, 98.94% of the students are expected to be serviced in 6 minutes or less (t=0.1), 99.97% in 12 minutes or less (t=0.2), so on and 100% of the students to be serviced in 36 minutes (t=.6).

Table 2.2 Service time distribution ofassumed values of service time (in hours)

X=SERVICE TIME (IN HOURS)	P(SERVICE TIME ≤X)
0.1	0.983427279
0.2	0.999725345
0.3	0.999995448
0.4	0.999999925
0.5	0.999999999
0.6	1
0.7	1
0.8	1
0.9	1
1.0	1

Operating Characteristics of Students
Serviced at the Cashier's Counter

Using the Poisson arrivals and exponential service times identical servers and a first come first serve discipline, operating characteristics for the multiserver-multiqueue-single-channel waiting line model was presented in Table 3.

Table 3 presents the teller utilization rate which is the percentage of the time a counter is busy. The total service rate must be greater than the arrival ratet that is $k\mu > \lambda$. If $s\mu \leq$ the average number of customer spends in the system (L) and the average time a student spends in the system (W) both become infinitely large. In table 3, the utilization rate is clear that the formula is applicable since $\lambda/k\mu < 1$ thus the probability that the servers were busy is 97% . the probability that the cashier is idle is .5%. This implies that the server in free for .3 minutes upon the start of the operation thus most of the time the cashier is busy. On the average, there are 39 students in the system. The average time a customer spends in the system is .51 hour or 30 minutes. The average time a customer spends in the queue before being served is .47 hour or 28 minutes. The average number of students in the queue is 36 students and the probability that an arriving customer has to wait for service is .95.

Suppose another counter is added with the same arrival rate and service rate and a first in-first-out discipline. Based on table 3 the probability that the servers are busy is reduced by 24% from 97% to 73%. The probability that the server is idle is increased from .5% to 3.4% or from .3 minutes to 2.09 minutes.

Table 3. Economic analysis on the operating characteristics of the queueing of students serviced at the cashiers counter

SERVERS (s)	$\rho=\lambda/s\mu$	P(0)	L	W	Lq	Wq	Pw
3	0.974358	0.00586	39	0.5142	36	0.475738	0.95125
4	0.730769	0.03489	14	0.17925	11	0.14078	0.3942

where:
L= average number of units in the system
W= the average time a unit spends in the system (waiting time + service)
Lq= the average number of units in the ueue waiting for service
Wq= the average time a unit spends in the queue waiting for service
Pw= the probability that an arriving students has to wait for service

The average number of student in the system is reduced by 25 from 39 to 14 students. The average time a student spends in the system is reduced by 20 minutes from .5142 hour to .17925 hour or from 30 minutes to 10 minutes. The average number of students in the queue is reduced by 25 from 36 to 11 students. The probability that an arriving customer will wait is reduced from 95% to 39%.

The assumption is clear that four counters will greatly improve operating characteristics of the system.

SUMMARY, CONCLUSION AND RECOMMENDATION

Summary

Poisson and exponential distribution were employed in this study. The average number of students who arrive is 76 students per hour and the average students that the cashier counter can service per hour were 26, that results to long lines of students.

The model was identified to analyze the enrollment operation for there are three cashier counters in the administration. The measuring capacity utilization (ρ) shows th97.44% of time the counters are busy. The average time a student spent at the system is .5142 hours, which means that a customer will receive complete service for approximately 30.85 minutes including the time the students waiting in the line. The average time in the queue is 0.475738 hours which is approximately 28.54 minutes, that is the time spent waiting in the line. The remaining time, which is W-Wq is the time spent in the counter, 0.038462 hours or 2.30772 minutes.

Conclusion

It is the variability in arrival and service distribution that causes waiting lines. Waiting line model allows to estimate performance by predicting average system utilization, average number of students in the service system, average number of students in the waiting line, average time a student spends in the system and the average time students wait in line.

The result on the operating characteristics indicates that students wait an average of .5142 hours or 31 minutes. Which appears to be undesirable for students to wait for their turn for almost half an hour. And shows that long waits suggest a lack of concern by the management or, can be linked to a perception of poor service management. But sometimes, it’s the students fault why they were served that long because they arent following the schedule given by the management for them to enroll.

Recommendation

The operating characteristics of the queueing of students serviced at the cashier’s counter suggest the management to change the arrival rates in a number of ways, such as providing discounts for the early enrollees. Adding one counter will help reduce a number of students waiting in long queues. Results show that if another counter is added, the average number of students in the system is reduced from 39 to 14 students per hour; the average time students spends in the system is reduced from 30.8 minutes to 10.75 minutes and the probability that an arriving student has to wait for service is reduced from 95% to 39%. The more the counter, the better the service is.

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